

ELECTROPRODUCTION OF RESONANCES AT LARGE MOMENTUM TRANSFERS

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Measurements of transition form factors for the electro-excitation of nucleon resonances in the $Q^2 = 5 - 14 \text{ GeV}^2$ region can provide one with the information on quark wave functions at small transverse separations. In particular a comparison of form factors for the states of opposite parity can give insight in the mechanisms of chiral symmetry breaking. I discuss perspectives of the theoretical description of such reactions using a combination of lattice calculations and light-cone sum rules.

Keywords: electroproduction; nucleon resonances; lattice QCD

1. Introduction

Electroproduction of nucleon resonances has long been recognized as an important tool in the exploration of the nucleon structure at different scales. There is a growing consensus that perturbative QCD (pQCD) factorization based on hard gluon exchange is not reached at present energies; however, the emergence of quarks and gluons as the adequate degrees of freedom is expected to happen earlier, at $Q^2 \sim$ a few GeV^2 . Measurements of the form factors in this transition region are planned at Jefferson Lab^{1,2} using the CLAS12 detector. In this talk such reactions are addressed from the theory perspective. I try to formulate the physics goals and explain an approach to the electroproduction of resonances that combines lattice calculations of wave functions at small transverse separations with dispersion relations and quark-hadron duality, known as light-cone sum rules (LCSR).

Quantum chromodynamics (QCD) predicts³⁻⁶ that at large momentum transfer the transition form factors become increasingly dominated by the contribution of the valence Fock state with small transverse separation between the partons. In reality the dominance of valence configurations is

likely to be achieved but “shrinkage” to small transverse separations does not seem to occur and also power counting rules based on helicity conservation do not work at moderate Q^2 that we are able to study experimentally.

To be more specific, the nucleon valence state contains contributions with different orbital angular momentum $\ell_z = 0, \pm 1$. The leading contribution to form factors at large Q^2 comes from the $\ell_z = 0$ state:⁶

$$|P \uparrow\rangle^{\ell_z=0} = \int \frac{[dx][d^2\vec{k}]}{6\sqrt{x_1 x_2 x_3}} \psi^{\ell_z=0}(x_i, \vec{k}_i) \times \\ \times \left\{ |u^\uparrow(x_1, \vec{k}_1) u^\downarrow(x_2, \vec{k}_2) d^\uparrow(x_3, \vec{k}_3)\rangle - |u^\uparrow(x_1, \vec{k}_1) d^\downarrow(x_2, \vec{k}_2) u^\uparrow(x_3, \vec{k}_3)\rangle \right\}$$

where $\psi^{\ell_z=0}(x_i, \vec{k}_i)$ is the three-quark light-cone wave function that depends on quark momentum fractions x_i and transverse momenta \vec{k}_i ^a.

The simplification that occurs at asymptotically large Q^2 is that the \vec{k} -dependence of wave functions becomes irrelevant and all necessary (non-perturbative) information is contained in the integral over transverse momenta

$$\Phi_3(x_i; \mu) = \int^\mu [d^2\vec{k}] \psi^{\ell_z=0}(x_i, \vec{k}_i)$$

where the cutoff $\mu \sim Q$ has to be imposed to make the integral converge.

The function $\Phi_3(x_i; \mu)$ is called the leading-twist distribution amplitude (DA). It can be studied using the operator product expansion⁷

$$\Phi_3(x_i; \mu) = 120 f_N(\mu) x_1 x_2 x_3 \left\{ 1 + c_{10}(\mu)(x_1 - 2x_2 + x_3) + c_{11}(\mu)(x_1 - x_3) \right. \\ \left. + c_{20}(\mu) [1 + 7(x_2 - 2x_1 x_3 - 2x_2^2)] + c_{21}(\mu) (1 - 4x_2)(x_1 - x_3) \right. \\ \left. + c_{22}(\mu) [3 - 9x_2 + 8x_2^2 - 12x_1 x_3] + \dots \right\} \quad (1)$$

where $f_N(\mu)$ (wave function at the origin) and $c_{ik}(\mu)$ (shape parameters) are scale-dependent coefficients which can be defined as matrix elements of (multiplicatively renormalizable) local operators. They can be calculated using QCD sum rules⁸ or lattice QCD.⁹ The DA $\Phi_3(x_i; \mu)$ is thus a much simpler object compared to the full light-cone wave function $\psi^{\ell_z=0}(x_i, \vec{k}_i)$; unfortunately this reduction does not seem to work in the $Q^2 = 5 - 15 \text{ GeV}^2$ range.

Another problem is that the standard pQCD factorization approach does not take into account contributions of states with non-vanishing orbital

^aHere and below we do not show contributions which vanish in the limit of small transverse separations, cf. Ref. [10].

angular momentum. For example¹⁰

$$|P \uparrow\rangle^{\ell_z=1} = \int \frac{[dx][d^2\vec{k}]}{6\sqrt{x_1x_2x_3}} \left[k_1^+ \psi_1^{\ell_z=1}(x_i, \vec{k}_i) + k_2^+ \psi_2^{\ell_z=1}(x_i, \vec{k}_i) \right] \times \\ \times \left\{ |u^\uparrow(x_1, \vec{k}_1)u^\downarrow(x_2, \vec{k}_2)d^\downarrow(x_3, \vec{k}_3)\rangle - |d^\uparrow(x_1, \vec{k}_1)u^\downarrow(x_2, \vec{k}_2)u^\downarrow(x_3, \vec{k}_3)\rangle \right\}$$

where $k^\pm = k_x \pm ik_y$. The new light-cone wave functions $\psi_1^{\ell_z=1}(x_i, \vec{k}_i)$ and $\psi_2^{\ell_z=1}(x_i, \vec{k}_i)$ are reduced to next-to-leading twist-4 nucleon DAs^{12,13}

$$\Phi_4(x_i; \mu) = \int^\mu \frac{[d^2\vec{k}]}{m_N x_3} k_3^- [k_1^+ \psi_1^{L=1} + k_2^+ \psi_2^{L=1}](x_i, \vec{k}_i) \\ \Psi_4(x_i; \mu) = \int^\mu \frac{[d^2\vec{k}]}{m_N x_2} k_2^- [k_1^+ \psi_1^{L=1} + k_2^+ \psi_2^{L=1}](x_i, \vec{k}_i)$$

and, again, can be studied using OPE^{7,12}

$$\Phi_4(x_i; \mu) = 12\lambda_1(\mu)x_1x_2 + 12f_N(\mu)x_1x_2 \left[1 + \frac{2}{3}(1 - 5x_3) \right] + \dots \\ \Psi_4(x_i; \mu) = 12\lambda_1(\mu)x_1x_3 + 12f_N(\mu)x_1x_3 \left[1 + \frac{2}{3}(1 - 5x_2) \right] + \dots \quad (2)$$

Note that to this accuracy twist-4 DAs include one new parameter only, λ_1 . A similar expansion and the reduction to DAs can be worked out for nucleon resonances. Electroproduction of nucleon resonances at high Q^2 gives access to three-quark wave functions, more precisely to the overlap integrals between the wave functions of the nucleon and the resonance. These overlap integrals are in general too complicated to be calculated in QCD directly and our strategy will be to reduce these integrals to convolutions of distribution amplitudes which can be constrained using lattice QCD.

We believe that one important physics goal for such studies will be to compare DAs (alias light-cone wave functions at small transverse separations) of baryon states with opposite parity, e.g. $J^P = 1/2^+$ and $J^P = 1/2^-$. It is well known that such “parity-doublets” are non-degenerate in QCD because of spontaneous breaking of chiral symmetry. It is not known, however, whether this difference mainly affects the “pion cloud” or it is present at short distances already and affects momentum fraction distributions of valence quarks. Moreover, there are indications¹⁴ that chiral symmetry is effectively restored in the spectra of higher-mass resonances and it would be extremely interesting to compare the corresponding wave functions.

The general strategy of combining the constraints on DAs from a lattice calculation with LCSRs to calculate the form factors is suggested in Ref. [15] where, as the first demonstration, we considered the electroproduction of

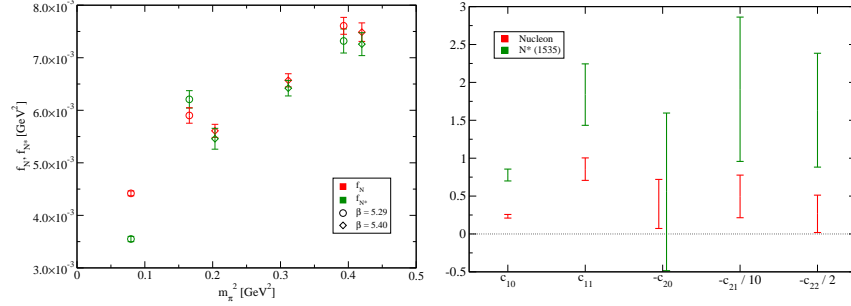


Fig. 1. Wave functions at the origin (left panel) and shape parameters of the nucleon and $N^*(1535)$ distribution amplitudes (right panel). Only statistical errors are shown.

$N^*(1535)$, the parity partner of the nucleon. In what follows I describe an ongoing work in this direction by the Regensburg theory group and QCDSF collaboration.

2. Nucleon and $N^*(1535)$ Distribution Amplitudes from Lattice QCD

Baryon states of different parity can be identified in a lattice calculation as those propagating forward or backward in (imaginary) time,^{16,17} so in fact the results for $N^*(1535)$ reported in Ref. [15] are essentially a byproduct of our calculation of the nucleon DAs.⁹ These results were obtained using QCDSF/DIK gauge configurations with two flavors of clover fermions for two different β values and several quark masses on $24^3 \times 48$ lattices. The calculation was done using nonperturbatively renormalized three-quark operators¹⁸ with up to two derivatives imposing a RI'-MOM-like renormalization condition and converting the results to the \overline{MS} scheme. In this way the mixing with “total derivatives” is automatically taken into account.

This work will be continued, using larger lattices and smaller pion masses in order to minimize effects of the chiral extrapolation. In Fig. 1 I present preliminary results¹⁹ of the new calculation using ca. 600 $N_f = 2$ gauge configurations on a $32^3 \times 64$ lattice with $\beta = 5.29$ ($a = 0.0753$ fm) and pion mass $m_\pi \simeq 282$ MeV ($m_\pi L = 3.44$). The left panel shows the comparison of nucleon and $N^*(1535)$ wave functions at the origin, f_N and f_{N^*} , as a function of m_π^2 . The left-most points at $m_\pi^2 \simeq 0.08$ GeV^2 are new. They are much closer to the physical point and show, for the first time, that for small pion masses f_{N^*} becomes smaller than f_N . This phenomenon still has to be understood.

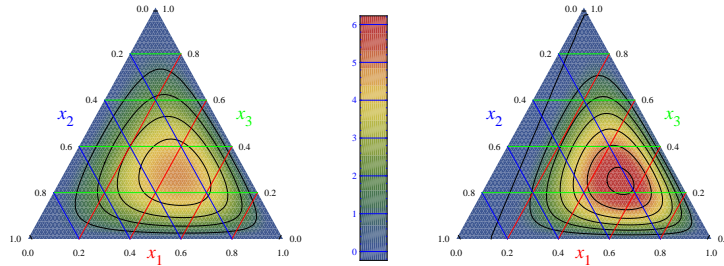


Fig. 2. Barycentric plots for the nucleon (left) and $N^*(1535)$ (right) distribution amplitudes.

The results for shape parameters of the nucleon distribution amplitudes (1) are shown on the right panel. One sees that coefficients of first-order polynomials c_{10} and c_{11} can be quantified, whereas contributions of second order polynomials are less constrained. The main reason for this are the $\mathcal{O}(a)$ discretization errors in the chain rule for derivatives $D(A \cdot B) = (DA) \cdot B + A \cdot (DB) + \mathcal{O}(a)$ which spoil energy conservation: after the (nonperturbative) renormalization we obtain for the sum of the quark momentum fractions $x_1 + x_2 + x_3 \simeq 0.94$ instead of unity. This is one of the issues that have to be addressed in future studies.

The main result so far is that the accuracy of modern lattice calculations is sufficient to detect differences in quark momentum fraction distributions in the nucleon and its parity partner state. Our calculations support a qualitative picture suggested by QCD sum rules⁸ that the valence quark with the spin parallel to that of the nucleon carries most of its momentum, and for $N^*(1535)$ this effect appears to be even stronger. As an illustration we show in Fig. 2 the DAs of the nucleon and $N^*(1535)$ in barycentric coordinates.^b Note that the $N^*(1535)$ DA appears to be more narrow and shifted towards the lower-right corner.

All these results are preliminary and the study will be continued using larger lattices and smaller pion masses. The comparison of the nucleon and $N^*(1535)$ will remain our primary goal for some time, but the calculations will also be extended to the full $J^P = 1/2^+$ and $J^P = 1/2^-$ baryon octets and to the decuplets. The main problem that has to be addressed in fu-

^bIn difference to the similar plot in Ref. [15] we only include the terms in c_{10} and c_{11} and discard contributions of second order polynomials which have large errors.

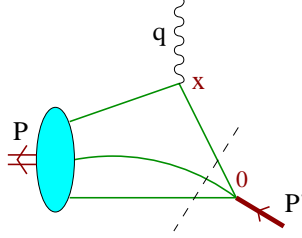


Fig. 3. Schematic structure of the light-cone sum rule for baryon form factors.

ture is the identification of resonance contributions for small quark (pion) masses such that strong decays, e.g. $N^*(1535) \rightarrow N\pi, N\eta$, are allowed. The separation of $N^*(1535)$ and $N^*(1650)$ may prove to be difficult. In particular the large decay width of $N^*(1535)$ in the $N\eta$ channel has to be understood and it can only be addressed in lattice calculations with three flavors of dynamic fermions. Discretization errors $\mathcal{O}(a)$ in the lattice definition of the relevant operators become a serious issue for second moments and have to be reduced in order that the results for c_{2k} coefficients become fully quantitative.

3. Transition form factors from Light-Cone Sum Rules

The matrix element of the electromagnetic current j_ν^{em} between spin-1/2 states of opposite parity can be parametrized in terms of two independent form factors, which can be chosen as

$$\begin{aligned} \langle N^*(P') | j_\nu^{\text{em}} | N(P) \rangle &= \bar{u}_{N^*}(P') \gamma_5 \Gamma_\nu u_N(P), \\ \Gamma_\nu &= \frac{G_1(q^2)}{m_N^2} (\not{q} q_\nu - q^2 \gamma_\nu) - i \frac{G_2(q^2)}{m_N} \sigma_{\nu\rho} q^\rho, \end{aligned} \quad (3)$$

where $q = P' - P$ is the momentum transfer. The LCSR are derived from the correlation function

$$\int dx e^{-iqx} \langle N^*(P) | T \{ \eta(0) j_\mu^{\text{em}}(x) \} | 0 \rangle,$$

where η is a suitable operator with nucleon quantum numbers, e.g. the Ioffe current.²⁰ Making use of the duality of QCD quark-gluon and hadronic degrees of freedom through dispersion relations one can write a representation for the form factors appearing in (3) in terms of the DAs of N^* . Schematically, the sum rules take the form

$$G_{1,2}(Q^2) = \sum_k \int [dx] C_{1,2;k}(x_i, Q^2, s_0, M^2, \mu, \alpha_s(\mu)) \Phi_k(x_i, \mu)$$

where the sum goes over contributions of nucleon DAs of increasing twist and C_k are the coefficient functions that can be calculated in QCD perturbation theory and modified using dispersion relations to include two nonperturbative parameters: s_0 , the interval of duality, and M^2 , the Borel parameter which specifies the distance (in imaginary time) on which matching between hadronic and quark representations for the correlation function is being done. The dependence on M^2 is unphysical in the same sense as the factorization scale μ dependence in truncated perturbative expansions, but it is usually weak. The pQCD limit³⁻⁶ corresponds to the leading part of the contribution of leading-twist DAs at large momentum transfer, and the main difference is that higher-twist contributions are *not* suppressed by powers of $\Lambda_{\text{QCD}}^2/Q^2$, but rather of $\Lambda_{\text{QCD}}^2/s_0$ with $s_0 \sim (1.5 \text{ GeV})^2$. The reason for this is that LCSRs take into account “soft” contributions to the form factors coming from large transverse separations. The attractive feature of this approach is that such terms are calculated in terms of the DAs, thus avoiding the need to know (model) the full nonperturbative k_\perp dependence of wave functions.

In leading order, the sum rules for $Q^2 G_1(Q^2)/(m_N m_{N^*})$ and $-2G_2(Q^2)$ have the same functional form as the similar sum rules^{21,22} for the Dirac and Pauli electromagnetic form factors of the proton, with the replacement $m_N \rightarrow m_{N^*}$ in the light-cone expansion part, and different DAs.

The experimental results for the electroproduction of $N^*(1535)$ are usually presented for helicity amplitudes $A_{1/2}(Q^2)$ and $S_{1/2}(Q^2)$ which can be expressed in terms of the form factors:²³

$$A_{1/2} = e B \left[Q^2 G_1(Q^2) + m_N (m_{N^*} - m_N) G_2(Q^2) \right],$$

$$S_{1/2} = \frac{e}{\sqrt{2}} B C \left[(m_N - m_{N^*}) G_1(Q^2) + m_N G_2(Q^2) \right].$$

Here e is the elementary charge and B, C are kinematic factors defined as

$$B = \sqrt{\frac{Q^2 + (m_{N^*} + m_N)^2}{2m_N^5(m_{N^*}^2 - m_N^2)}}, \quad C = \sqrt{1 + \frac{(Q^2 - m_{N^*}^2 + m_N^2)^2}{4Q^2 m_{N^*}^2}}.$$

The results of the LCSR calculation of the form factors (normalized to the dipole) and helicity amplitudes using lattice-constrained N^* DAs from Ref. [15] is presented in Fig. 4. The shaded areas show the estimated uncertainties. The Q^2 -dependence of the form factors $Q^2 G_1(Q^2)$ and $G_2(Q^2)$ is predicted to be similar to the Dirac and Pauli nucleon electromagnetic form factors, respectively. Only the normalization is different: The observed negative amplitude $S_{1/2}$ implies a small value of G_2 .

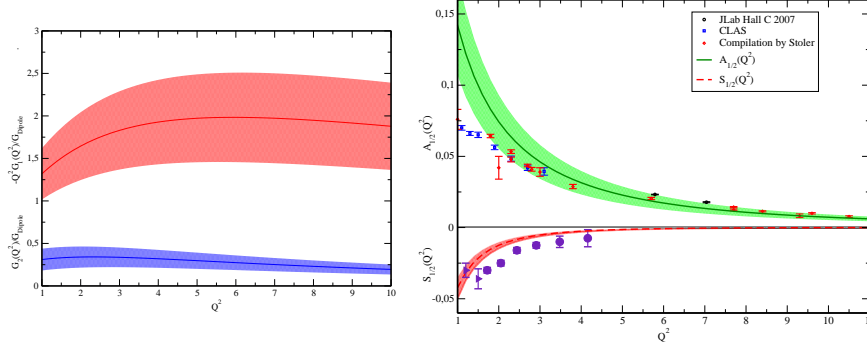


Fig. 4. Form factors (left panel) and helicity amplitudes (right panel) for the electroexcitation of the $N^*(1535)$ resonance calculated using the DAs in Ref. [15]. The experimental data are from Refs. [24–27].

These results have to be viewed as exploratory and further work is needed to make them quantitative. First of all, the LCSRs for baryon form factors have to be extended to the next-to-leading (NLO) accuracy, i.e. including the $\mathcal{O}(\alpha_s)$ corrections. This calculation is rather cumbersome and not straightforward because of contributions of evanescent operators. As the first step in this direction, the NLO corrections to contributions of leading-twist DAs in the LCSRs for the electromagnetic nucleon form factors F_1 and F_2 have been calculated in Ref. [28], see Fig. 5. One sees that such corrections are significant, especially in the G_E/G_M ratio. The extension of this calculation to contributions of sub-leading twist-4 DAs and the detailed study of baryon mass corrections to the sum rules is planned for the coming years.

Second, one has to remember that the LCSRs suffer from irreducible uncertainties due to the duality assumption for contributions of higher resonances and the continuum which for meson form factors is estimated to be of order 10%. (see e.g. Ref. [29]). The experience with applying this method to baryons is much less than for mesons, so that this “systematic error” cannot be estimated reliably. It is therefore imperative to apply the same technique to a maximally broad class of reactions. At present applications include nucleon electromagnetic and axial form factors,^{21,22,30} $N\gamma\Delta$ transitions,³³ pseudoscalar- and vector-meson couplings to octet and decuplet baryons,³⁴ weak decays of the type $\Lambda_b \rightarrow p\ell\nu_\ell$ ³¹ and $\Lambda_b \rightarrow \Lambda\ell^+\ell^-$,³² and pion electroproduction at threshold.^{35,36} I expect that this list will continue to grow, giving us confidence in the whole program.

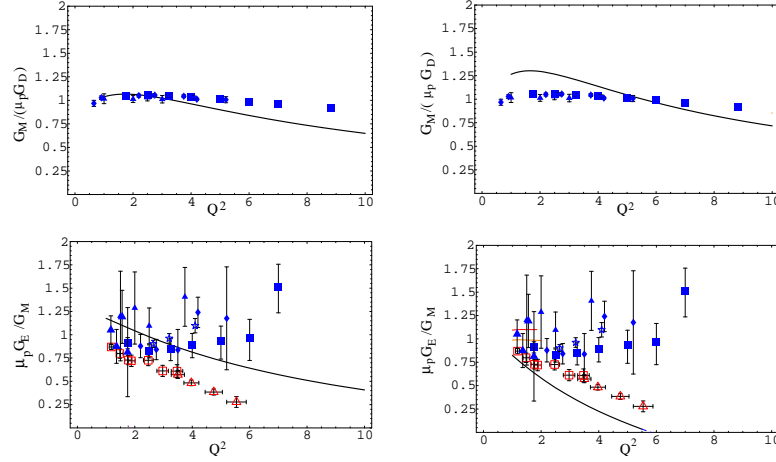


Fig. 5. LCSR results for the electromagnetic proton form factors for a realistic model of nucleon distribution amplitudes. Left panel: Leading order (LO); right panel: next-to-leading order (NLO) for twist-three contributions. Figure adapted from Ref. [28].

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